Bull. Math. Soc. Sci. Math. Roumanie
Tome 52(100) No. 3, 2009, 347–353

On the local structure of generalized Kähler manifolds

by
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To Professor S. Ianuş on the occasion of his 70th Birthday

Abstract

Let (g, b, J_+, J_-) be the bihermitian structure corresponding to a generalized Kähler structure. We find natural integrability conditions, in terms of the eigendistributions of $J_+J_- + J_-J_+$, under which $\mathrm{d}b = 0$.

Key Words: generalized Kähler manifold
2000 Mathematics Subject Classification: Primary 53C55, Secondary 53C26.

Introduction

A *generalized almost complex structure* on a smooth (connected) manifold is given by a vector subbundle $L \subset \big(TM \oplus T^*M\big)^{\mathbb{C}}$ such that $L \cap \overline{L} = \{0\}$ and which is maximally isotropic with respect to the canonical inner product

$$<X + \alpha, Y + \beta>= \tfrac{1}{2}\big(\alpha(Y) + \beta(X)\big) .$$

If $E = \pi_{TM}(L)$ is a bundle, where $\pi_{TM} : TM \oplus T^*M \to TM$ is the projection, then there exists a unique complex two-form $\varepsilon \in \Gamma(\Lambda^2 E^*)$ such that $L = L(E, \varepsilon)$, where

$$L(E, \varepsilon) = \{X + \alpha \,|\, X \in E, \alpha|_E = \varepsilon(X)\} .$$

Furthermore, by [4], to which we refer for all of the facts on generalized complex structures recalled here, the condition $L \cap \overline{L} = \{0\}$ is equivalent to $E + \overline{E} = T^{\mathbb{C}}M$ and $\mathrm{Im}\big(\varepsilon|_{E\cap\overline{E}}\big)$ is non-degenerate.

A generalized almost complex structure L is *integrable* if its space of sections is closed under the *Courant bracket*, defined by

$$[X + \alpha, Y + \beta] = [X, Y] + \mathcal{L}_X\beta - \mathcal{L}_Y\alpha - \tfrac{1}{2}\,\mathrm{d}(\iota_X\beta - \iota_Y\alpha) ,$$

for any $X + \alpha, Y + \beta \in \Gamma(L)$.

A *generalized complex structure* is an integrable generalized almost complex structure. Obviously, any generalized complex structure corresponds to a linear complex structure on $TM \oplus T^*M$ whose eigenbundle, corresponding to i, is isotropic, with respect to the canonical inner product, and its space of sections is closed under the Courant bracket.

A generalized almost complex structure of the form $L = L(E, \varepsilon)$ is integrable if and only if the

*The authors are partially supported by a PN II IDEI Grant, code 1193.

space of sections of E is closed under the (Lie) bracket and $\mathrm{d}\varepsilon(X,Y,Z)=0$, for any $X,Y,Z\in E$.

A particular feature of Generalized Complex Geometry is that imposing Hermitian compatibility to a generalized almost complex structure and a Riemannian metric on $TM\oplus T^*M$, compatible with the canonical inner product, forces the manifold to admit a second generalized almost complex structure, commuting with the first one. One arrives to the notion of *generalized Kähler structure*, as a couple of commuting generalized complex structures $\mathcal{J}_1$ and $\mathcal{J}_2$ such that $\mathcal{J}_1\mathcal{J}_2$ is negative definite; furthermore, in [4] it is explained the correspondence between generalized Kähler structures and a special type of bihermitian structures which appeared in Theoretical Physics, over twenty years ago [3].

More precisely, any generalized almost Kähler structure on M corresponds to a quadruple (g,b,J_+,J_-), where g is a Riemannian metric, b is a two-form and $J_\pm$ are almost Hermitian structures on (M,g). Furthermore, the corresponding generalized almost Kähler structure is integrable if and only if $J_\pm$ are integrable and parallel with respect to $\nabla^\pm$, where $\nabla^\pm=\nabla\pm\frac{1}{2}g^{-1}h$, with ∇ the Levi-Civita connection of g and $h=\mathrm{d}b$ (equivalently, $J_\pm$ are integrable and $\mathrm{d}^c_\pm\omega_\pm=\mp h$, where $\omega_\pm$ are the Kähler forms of $J_\pm$).

Classification results for compact bihermitian manifolds were given, mainly in dimension 4, in several papers (see, for example, [1], [2]).

In higher dimensions, a natural case to consider is when J_+ and J_- are admissible for an almost quaternionic structure. This condition was, essentially, considered by physicists who have shown that it holds if and only if the bihermitian structure is part of a hyperkähler one [7, Theorem 1] (see Theorem 1.1, below).

By combining this fact with results of [9] and [8], we study the 'eigendistributions' of the operator $J_+J_-+J_-J_+$. Thus, we obtain natural integrability conditions under which $\mathrm{d}b=0$ (Theorem 2.3, Corollary 2.4).

1 The almost quaternionic generalized Kähler manifolds are hyperkähler

A *bundle of associative algebras* is a vector bundle whose typical fibre is an associative algebra $\mathcal{A}$ and whose structural group is the group of automorphisms of $\mathcal{A}$.

An *almost quaternionic structure* on M is a morphism of bundles of associative algebras $\sigma:A\to\mathrm{End}(TM)$, where the typical fibre of A is $\mathbb{H}$. Then, $\sigma(\mathrm{Im}A)$ is an oriented Riemannian vector bundle of rank 3 and the (local) sections of its sphere bundle are the *admissible almost complex structures* of σ (see [6]).

The following result reformulates [7, Theorem 1]. For the reader's convenience, we supply a proof.

Theorem 1.1. *Let (M,L_1,L_2) be a generalized almost Kähler manifold of dimension at least eight and let (g,b,J_+,J_-) be the corresponding almost bihermititan structure. Suppose that J_+ and J_- are admissible almost complex structures of an almost quaternionic structure on M.*

Then the following assertions are equivalent:

(i) *(M,L_1,L_2) is generalized Kähler.*

(ii) *$(M,g,J_\pm)$ are Kähler manifolds.*

Furthermore, if (i) *or* (ii) *holds and $J_+\neq\pm J_-$ then the almost quaternionic structure is hyperkähler, with respect to (M,g).*

Proof: As (ii)$\Longrightarrow$(i) is trivial, it is sufficient to prove that (i)$\Longrightarrow$(ii).

By hypothesis, there exists $a:M\to[-1,1]$ such that $J_+J_-+J_-J_+=-2a$ on M. If $J_+=\pm J_-$ there is nothing to be proved. Hence, we may suppose that $a^{-1}\big((-1,1)\big)\neq\emptyset$.

Moreover, as we have to prove that $(M,g,J_\pm)$ are Kähler and, consequently, a is constant, we may assume $a(M)\subseteq(-1,1)$.

Then $L_1 = L(T^{\mathbb{C}}M, \varepsilon_+)$ and $L_2 = L(T^{\mathbb{C}}M, \varepsilon_-)$, where $\varepsilon_\pm$ are closed complex two-forms on M. From [4, (6.4) and (6.5)], it quickly follows that

$$\begin{aligned}(\mathrm{Im}\,\varepsilon_\pm)(J_+ \mp J_-) &= 2g\,,\\ (\mathrm{Re}\,\varepsilon_\pm)(J_+ \mp J_-) &= b(J_+ \mp J_-) + g(J_+ \pm J_-)\,.\end{aligned} \tag{1.1}$$

On multiplying, to the right, both relations of (1.1) by $J_+ \mp J_-$ we obtain

$$\begin{aligned}(-2 \pm 2a)(\mathrm{Im}\,\varepsilon_\pm) &= 2g(J_+ \mp J_-)\,,\\ (-2 \pm 2a)(\mathrm{Re}\,\varepsilon_\pm) &= (-2 \pm 2a)b \mp g(J_+J_- - J_-J_+)\end{aligned}$$

and, consequently, $(a-1)\mathrm{Re}\,\varepsilon_+ - (a+1)\mathrm{Re}\,\varepsilon_- = -2b$.

Therefore

$$\mathrm{d}\left[\frac{1}{1 \pm a} g(J_+ \pm J_-)\right] = 0\,. \tag{1.2}$$

Also, as, up to a B-field transformation, we may suppose $\mathrm{Re}\,\varepsilon_- = 0$, we deduce that the two-form $\frac{1}{a-1}b$ is closed; equivalently,

$$\mathrm{d}b = \frac{1}{a-1}\,\mathrm{d}a \wedge b\,. \tag{1.3}$$

Note that, the condition $\nabla^\pm J_\pm = 0$ is equivalent to

$$g\big((\nabla_X J_\pm)(Y), Z\big) = \mp\tfrac{1}{2}\big[(\mathrm{d}b)(X, J_\pm Y, Z) + (\mathrm{d}b)(X, Y, J_\pm Z)\big]\,, \tag{1.4}$$

for any $X, Y, Z \in TM$.

From (1.3) and (1.4) we obtain

$$g\big((\nabla_X J_\pm)(Y), Z\big) = \pm\frac{1}{2(1-a)}\big(\mathrm{d}a \wedge b\big)\big(X \wedge J_\pm Y \wedge Z + X \wedge Y \wedge J_\pm Z\big)\,, \tag{1.5}$$

for any $X, Y, Z \in TM$.

Obviously,

$$K_\pm = \frac{1}{\sqrt{2(1 \pm a)}}\big(J_+ \pm J_-\big)\,.$$

are anti-commuting almost Hermitian structures on (M, g). Furthermore, (1.5) gives

$$\begin{aligned}g\big((\nabla_X K_\pm)(Y), Z\big) &= \mp\frac{1}{2(1 \pm a)}\,X(a)\,g(K_\pm Y, Z)\\ &+ \frac{1}{2(1-a)}\left(\frac{1-a}{1+a}\right)^{\pm\frac{1}{2}}\big(\mathrm{d}a \wedge b\big)\big(X \wedge K_\mp Y \wedge Z + X \wedge Y \wedge K_\mp Z\big)\,,\end{aligned} \tag{1.6}$$

for any $X, Y, Z \in TM$.

On the other hand, by (1.2), the almost Hermitian manifolds $\big(M, e^{2f_\pm}g, K_\pm\big)$ are $(1,2)$-symplectic, where $f_\pm = -\frac{1}{4}\log 2(1 \pm a)$. A straightforward calculation shows that this is equivalent to

$$\begin{aligned}g\big((\nabla_{K_\pm X} K_\pm)(Y), Z\big) - g\big((\nabla_X K_\pm)(Y), K_\pm Z\big) &=\\ \pm\frac{1}{2(1 \pm a)}\big[(K_\pm Y)(a)\,g(K_\pm X, Z) - (K_\pm Z)(a)\,g(K_\pm X, Y)&\\ + Y(a)\,g(X, Z) - Z(a)\,g(X, Y)\big]\,,&\end{aligned} \tag{1.7}$$

for any $X, Y, Z \in TM$.

Now, (1.6) and (1.7) imply

$$\begin{aligned}(K_{\pm}X)(a)\,g(K_{\pm}Y,Z) + (K_{\pm}Y)(a)\,g(K_{\pm}X,Z) - (K_{\pm}Z)(a)\,g(K_{\pm}X,Y)\\ -X(a)\,g(Y,Z) + Y(a)\,g(X,Z) - Z(a)\,g(X,Y) =\\ \pm\left(\frac{1-a}{1+a}\right)^{-\frac{1}{2}}\big(\mathrm{d}a\wedge b\big)\big(K_{\pm}X\wedge K_{\mp}Y\wedge Z + K_{\pm}X\wedge Y\wedge K_{\mp}Z\\ - X\wedge K_{\mp}Y\wedge K_{\pm}Z - X\wedge Y\wedge K_{\mp}K_{\pm}Z\big)\,,\end{aligned} \tag{1.8}$$

for any $X, Y, Z \in TM$.

In (1.8) , if from the first relation we subtract the second one, with the roles of X and Y interchanged, then we obtain

$$\begin{aligned}&(K_{+}X)(a)\,g(K_{+}Y,Z) + (K_{+}Y)(a)\,g(K_{+}X,Z) - (K_{+}Z)(a)\,g(K_{+}X,Y)\\ &+ (K_{-}X)(a)\,g(K_{-}Y,Z) + (K_{-}Y)(a)\,g(K_{-}X,Z) + (K_{-}Z)(a)\,g(K_{-}X,Y)\\ &- 2Z(a)\,g(X,Y) = 2\left(\frac{1-a}{1+a}\right)^{-\frac{1}{2}}\big(\mathrm{d}a\wedge b\big)\big(K_{+}X\wedge K_{-}Y\wedge Z\big)\,,\end{aligned} \tag{1.9}$$

for any $X, Y, Z \in TM$.

From (1.9) , with $Z = K_{+}X$, it quickly follows that $\mathrm{grad}_g a$ is zero on the orthogonal complement of each quaternionic line. As $\dim M \geq 8$, we obtain that a is constant. Together with (1.6) , this gives that $K_{\pm}$ generate a hyperkähler structure on (M, g) , whilst, together with (1.3) , this implies $\mathrm{d}b = 0$. The proof is complete. □

Remark 1.2. In dimension four, the hypothesis of Theorem 1.1 is equivalent to the condition that J_{+} and J_{-} induce the same orientation on M, whilst if J_{+} and J_{-} induce different orientations on M then, up to a unique B-field transformation, M is locally given by a product of two Kähler manifolds (consequence of [8, Corollary 5.7]). Furthermore, there exist four-dimensional generalized Kähler manifolds with J_{+} and J_{-} inducing the same orientation and which are not given by a hyperkähler structure (see [5]).

The next result follows quickly from (1.3) and (1.9) .

Corollary 1.3. *Let (M, L_1, L_2) be a four-dimensional generalized Kähler manifold with J_{+} , J_{-} inducing the same orientation on M and linearly independent, at each point.*

Then, up to a unique B-field transformation, the following relations hold:

$$\begin{aligned}\mathrm{d}b &= -\,\frac{1}{1-a}\,\mathrm{d}a\wedge b\,.\\ *(\mathrm{d}a\wedge b) &= \frac{1}{2(1+a)}\,[J_{+},J_{-}](\mathrm{d}a)\,,\end{aligned} \tag{1.10}$$

where $*$ is the Hodge star operator of (M, g) and the function $a : M \to (-1, 1)$ is characterised by $J_{+}J_{-} + J_{-}J_{+} = -2a$.

We end this section by showing how equations (1.10) can be slightly simplified.

Remark 1.4. Let (M, L_1, L_2) be a four-dimensional generalized Kähler manifold with J_{+} , J_{-} inducing the same orientation on M and linearly independent, at each point.

With the same notations as in Theorem 1.1, let $K = K_{+}K_{-}$, $k = \left(\frac{1+a}{1-a}\right)^{\frac{1}{2}} g$ and $u = \log(1-a)$.

Then (1.10) is equivalent to

$$\mathrm{d}b = \mathrm{d}u\wedge b = -\,*_k K\mathrm{d}u\,. \tag{1.11}$$

If $\mathrm{d}u$ is nowhere zero, then the second equality of (1.11) is equivalent to

$$b = cv_{\mathscr{E}} + v_{\mathscr{F}} + \mathrm{d}u \wedge \alpha \,,$$

where c is a function, $\mathscr{E}$ is generated by $\{\operatorname{grad} u, K(\operatorname{grad} u)\}$, $\mathscr{F} = \mathscr{E}^{\perp}$, α is a section of $\mathscr{F}^*$, and $v_{\mathscr{E}}$, $v_{\mathscr{F}}$ are the volume forms of $\mathscr{E}$, $\mathscr{F}$, respectively.

2 Factorisation results for generalized Kähler manifolds

Let (M, L_1, L_2) be a generalized Kähler manifold and let (g, b, J_+, J_-) be the corresponding bihermitian structure. For any $a \in [-1, 1]$, we (pointwisely) denote by $\mathscr{H}^a$ the eigenspace of $J_+J_- + J_-J_+$ corresponding to $-2a$; also, we denote $\mathscr{H}^{\pm} = \mathscr{H}^{\pm 1}$ and $\mathscr{V} = \big(\mathscr{H}^+ \oplus \mathscr{H}^-\big)^{\perp}$. Then, at each point of M, we have that $\mathscr{H}^a$ are preserved by $J_{\pm}$ and there exist (finite) orthogonal decompositions $TM = \bigoplus_a \mathscr{H}^a$ and $\mathscr{V} = \bigoplus_{|a|<1} \mathscr{H}^a$.

Corollary 2.1. *Let N be a complex submanifold of $(M, J_{\pm})$, of complex dimension at least four, endowed with a function $a : N \to (-1, 1)$ such that $T_xN \subseteq \mathscr{H}_x^{a(x)}$, $(x \in N)$.*

Then a is constant and N is endowed with a natural hyperkähler structure whose underlying Riemannian metric is $g|_N$ and for which $J_+|_N$ and $J_-|_N$ are admissible complex structures.

Proof: As, obviously, (g, b, J_+, J_-) induces a generalized Kähler structure on N, this follows quickly from Theorem 1.1. □

From [9, Lemma 2.3] it follows that in an open neighbourhood U of each point of a dense open subset of M there exist (smooth) functions $a_j : M \to [-1, 1]$, $(j = 1, \dots, r)$, such that $\mathscr{H}^{a_j}$ are distributions on U and $TM = \bigoplus_j \mathscr{H}^{a_j}$; we call the $\mathscr{H}^{a_j}$ the *(local) eigendistributions* of $J_+J_- + J_-J_+$. Furthermore, if a is a function on U such that, at each point, $-2a$ is an eigenvalue of $J_+J_- + J_-J_+$ then there exists an open subset of U on which $a = a_j$, for some j; thus, if we assume real-analyticity then $a = a_j$ on U.

We point out the following facts:

- *The functions a_j are constant along the integrable manifolds, of dimensions at least eight, of* $\mathscr{H}^{a_j}$, $(j = 1, \dots, r)$; this is a consequence of Corollary 2.1.
- *If $J_+ \pm J_-$ are invertible then the holomorphic diffeomorphisms of (M, L_1, L_2) preserve each* $\mathscr{H}^{a_j}$, $(j = 1, \dots, r)$; this is a consequence of [8, Corollary 6.7].

Remark 2.2. Let (M, L_1, L_2) be a generalized Kähler manifold with $\mathrm{d}b = 0$. Then $(M, g, J_{\pm})$ are Kähler and there exists a nonempty finite subset A of $[-1, 1]$ such that, for any $a \in A$, we have that $\mathscr{H}^a$ is a parallel foliation which is holomorphic with respect to both J_+ and J_-. Therefore $(g, J_{\pm})$ induce Kähler structures on the leaves of $\mathscr{H}^a$ and, if $a \neq \pm 1$, these are admissible with respect to natural hyperkähler structures. Furthermore, there exist orthogonal decompositions $TM = \bigoplus_{a \in A} \mathscr{H}^a$ and $\mathscr{V} = \bigoplus_{a \in A \setminus \{\pm 1\}} \mathscr{H}^a$.

If the cardinal of $A \setminus \{\pm 1\}$ is at least two then the leaves of $\bigoplus_{a \in A \setminus \{\pm 1\}} \mathscr{H}^a$ are naturally endowed with two distinct hyperkähler structures with respect to which J_+ and J_- define admissible complex structures, respectively.

Furthermore, if $J_+ + J_-$ (or $J_+ - J_-$) is invertible then as, locally, M is the product of a Kähler manifold and hyperkähler manifolds, its holomorphic Poisson structure is the pull-back of the product of the holomorphic symplectic structures of the hyperkähler factors.

Next, we prove the following.

Theorem 2.3. *Let (M, L_1, L_2) be a generalized Kähler manifold with $J_+ + J_-$ (or $J_+ - J_-$) invertible and for which the eigendistributions of $(J_+J_- + J_-J_+)|_{(\mathscr{H}^+ \oplus \mathscr{H}^-)^\perp}$ have dimensions at least eight.*

Then the following assertions are equivalent:

(i) $\mathrm{d}b = 0$.

(ii) *The eigendistributions of $J_+J_- + J_-J_+$ and their orthogonal complements are integrable.*

Proof: The implication (i)$\Longrightarrow$(ii) is an immediate consequence of Remark 2.2.

Assume that (ii) holds. From [8, Corollary 6.3] it follows that we may suppose that, also, $J_+ - J_-$ is invertible.

Then, locally, outside a set with empty interior there exists a finite set A of functions $a : M \to (-1, 1)$ such that $\mathscr{H}^a$ are distributions and $TM = \bigoplus_{a \in A} \mathscr{H}^a$.

Also, $L_1 = L(T^{\mathbb{C}}M, \varepsilon_+)$ and $L_2 = L(T^{\mathbb{C}}M, \varepsilon_-)$, where $\varepsilon_\pm$ are closed complex two-forms on M.

By Theorem 1.1, we have that (i) holds if and only if $\mathrm{d}b(X, Y, Z) = 0$, for any $X \in \mathscr{H}^a$ and $Y, Z \in \bigoplus_{a' \in A \setminus \{a\}} \mathscr{H}^{a'}$, $(a \in A)$.

As $\mathscr{H}^a$, $(a \in A)$, are invariant under B-field transformations, we may assume $\mathrm{Re}\,\varepsilon_- = 0$; equivalently, $b = -g(J_+ - J_-)(J_+ + J_-)^{-1}$. Together with the fact that $\mathscr{H}^a$, $(a \in A)$, and their orthogonal complements are holomorphic foliations, with respect to J_+ and J_-, this gives that (i) holds if and only if $\mathscr{H}^a$ are Riemannian foliations, $(a \in A)$.

Now, note that we, also, have

$$\mathrm{Re}\,\varepsilon_+ = b + g(J_+ + J_-)(J_+ - J_-)^{-1} = g\big[(J_+ + J_-)(J_+ - J_-)^{-1} - (J_+ - J_-)(J_+ + J_-)^{-1}\big] .$$

As L_1 is integrable, $\mathrm{Re}\,\varepsilon_+$ is closed and, consequently, $\mathscr{H}^a$ are Riemannian foliations, $(a \in A)$.

The proof is complete. □

We end with the following result.

Corollary 2.4. *Let (M, L_1, L_2) be a generalized Kähler manifold for which the eigendistributions of $(J_+J_- + J_-J_+)|_{(\mathscr{H}^+ \oplus \mathscr{H}^-)^\perp}$ have dimensions at least eight.*

Then the following assertions are equivalent:

(i) $\mathrm{d}b = 0$.

(ii) *$\mathscr{H}^\pm$ and the sum of any two eigendistributions of $J_+J_- + J_-J_+$ are integrable.*

Proof: The implication (i)$\Longrightarrow$(ii) is trivial.

If (ii) holds then $\mathscr{H}^+ \oplus \mathscr{H}^-$ is integrable. Hence, by [8, Theorem 6.10], we may assume $\mathscr{H}^+ = 0 = \mathscr{H}^-$. The proof follows from Theorem 2.3. □

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Received: 20.04.2009.

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